Collision Planar Impact Analysis

This collision model is based on the ***Planar Impact Mechanics*** Model (PIM)[1] of Raymond M. Brach. The PIM model is a time forward calculation that inputs linear and angular velocites of two vehicles colliding at a point C along a normal surface at angle Γ, given two constraints for the **Coefficient of Restitution, ε**, and the **Impulse Ratio, μ.** PIM then solves this for the final velocities.

The coefficient of restitution is the ratio of the Final Normal RelativeVelocity at the common point o contact of the collision (C), normal to Γ, V_{crn}. to the initial normal relative velocity, v_{crn}. Then the coefficient of restitution, ε, equals $-V_{crn}/v_{crn}$. Equivalently, $\varepsilon = (V_{2n} - V_{1n})/(v_{2n} - v_{1n})$ or $\varepsilon = -Pr/P$

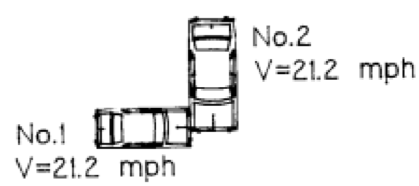
The Impulse ratio gives the **tangential common velocity condition**, that is, where $\mathbf{V_{1Ct} - V_{2Ct} = 0}$.
Let β be the fraction of μ_0 that is used for the reconstruction. Then $\mu = \beta * \mu_0$,
where β is a measure of the intervehicle sliding.

Goals of this Analysis:

Develop a Mathcad implementation and an explicit and deeper investigation of the PIM.
Express results as functions of ε and β, for example, as $V_{x1}(\varepsilon,\beta)$, for the final x-velocity of car 1.
We can then calculate and show the sensitivties of final results to the coefficient of restitution ε.
Find the Optimum values of ε and μ to minimize the Least Squares error of the final velocities.
Verify results by comparing against the RISCAC 9 collision and Fig 7.14[1].
This functional form allows the final results to be graphed as shown on pages 8 and 9.

Validation: Analysis of RICSAC Collision #9 Brach[1] Fig 7.14

The Research Input for Computer Simulation of Automobile Collisions (RICSAC) was a federally funded research project by CALSPAN Corporation's Advanced Technology Center. The intent was to provide critical data for validating existing and future computer analysis and simulation programs. There were 12 series of staged collsions configurations. Collision #9 was an oblique 90 degree collsion as illustrated below.

9	No. 1 - '74 Honda Civic No. 2 - '74 Ford Torino	Oblique	

$\varepsilon := 0.245$ $\mu := 0.714$ $e_m := -0.914$ $E_{loss\%} := 40$ $\Omega_{Honda} := -3.142 \cdot \frac{rad}{s}$ $\Omega_{Ford} := 7.86 \cdot \frac{rad}{s}$

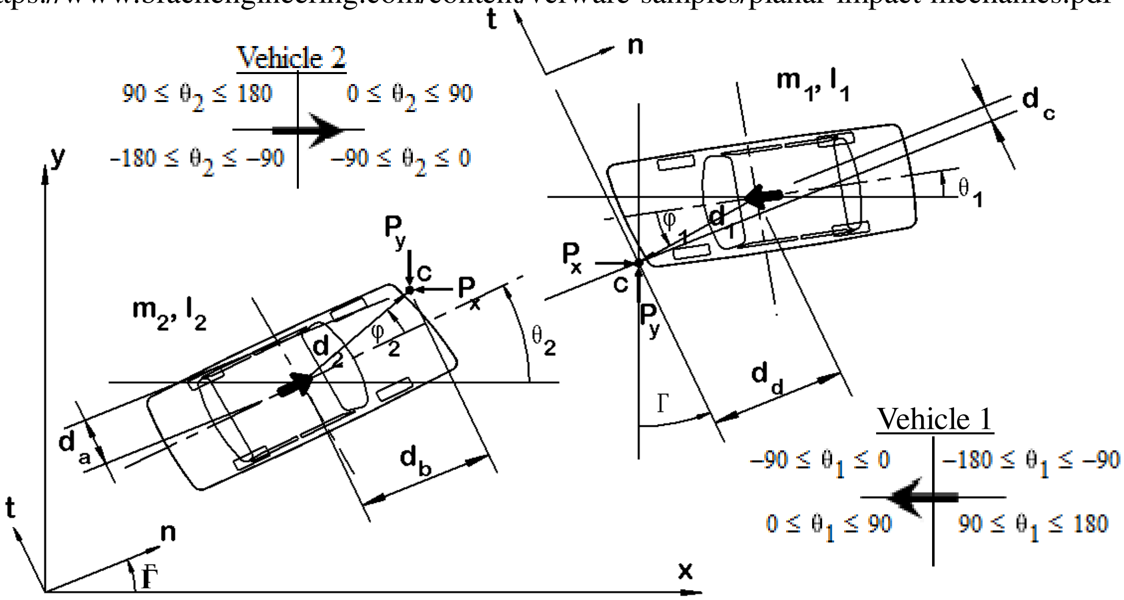
Note: Brach notation[1] has initial v_{x1} as negative, θ1 = 0 degrees, θ2 = 90 degrees, and Γ = 0 degrees.
Detailed RICSAC 9 input conditions in the PIM model are given in the table on the following page.
The orientation used in the Brach Fig 7.14 for RICSAC 9 differs from the above.

VXPhysics.com

PIM - Forward Calculation Conservation of Momentum Model

The drawing below shows the general relationships between the Brach variables

https://www.brachengineering.com/content/vcrware-samples/planar-impact-mechanics.pdf



Given:

1. Initial velocity components: v_{1x}, v_{1y}, v_{2x}, v_{2y}, ω, where ω is relative to x or street axis.

2. Vehicle physical properties: Weight_1 (mass_1), Weight_2 (mass_2, and Moments of Inertia: I_1, I_2

3. Heading orientation angles: θ_1, θ_2 (relative to x axes). All angles, except ϕ, are relative to x axis

4. Heading angles for **head on collision** have a datum of 0 degrees CCW for Vehicle1 moving to le

5. Collision damage characteristics of **common point C**: d_1, d_2, ϕ_1, ϕ_2, (ds to CM, ϕs relative to θs)

The contact surface (~ stiffness) is the time and space average of the deformed contact surface.
C is the common/impact center point of the intersection of the impulse and contact surface.
Energy loss: ε (coeff of normal restitution), The normal impulse ($P=\Delta p$) can be divided into 2 parts
Pa during approach and Pr during rebound, such that $\varepsilon = -\,Pr/Pa$, ε of kinetic coeff restitution
Equivalently, $\varepsilon = (V_{2n} - V_{1n})/(v_{2n} - v_{1n})$. Guess μ (ratio of tangential to normal impulse) Max μ_0.
A normal and tangential coordinate system is referenced wrt a common contact or crush surface.
Γ is the angle of the normal to the collision plane relative to the x or street axis. N like x, is + to Rig
Input conditions are given in lower case and Output parameters are upper case.

Find: Final velocities $\mathbf{V_{x1}, V_{x2}, V_{y1}}$, **and** $\mathbf{V_{y2}}$ and angular momenta, $\mathbf{\Omega_1, \Omega_2}$.

Initial Conditions:	**Vehicle 1**	**Vehicle 2**
Weight/Mass, w/m Moment of Inertia, I Common pt. moment arm, d Angle @CM of hdg θ to P, ϕ CCW Impulse "heading" angle, θ Initial v_x Initial v_y Angular velocity, ω	$\begin{pmatrix} m_1 \\ I_1 \\ d_1 \\ \phi_1 \\ \theta_1 \\ v_{x1} \\ v_{y1} \\ \omega_1 \end{pmatrix} := \begin{pmatrix} 70.1\,\text{lb}\cdot\text{s}^2\cdot\text{ft}^{-1} \\ 976\cdot\text{ft}\cdot\text{lb}\cdot\text{s}^2 \\ 4.8\cdot\text{ft} \\ 6 \\ 0 \\ -31.09\cdot\text{ft}\cdot\text{s}^{-1} \\ 0\,\text{mph} \\ 0\cdot\text{deg}\cdot\text{s}^{-1} \end{pmatrix}$	$\begin{pmatrix} m_2 \\ I_2 \\ d_2 \\ \phi_2 \\ \theta_2 \\ v_{x2} \\ v_{y2} \\ \omega_2 \end{pmatrix} := \begin{pmatrix} 152.2\,\text{lb}\cdot\text{s}^2\cdot\text{ft}^{-1} \\ 3953.3\cdot\text{ft}\cdot\text{lb}\cdot\text{s}^2 \\ 5.6\cdot\text{ft} \\ -29.7 \\ 90 \\ 0\cdot\text{ft}\cdot\text{s}^{-1} \\ 31.09\cdot\text{ft}\cdot\text{s}^{-1} \\ 0\,\text{deg}\cdot\text{s}^{-1} \end{pmatrix}$

Angle of normal to collision plane, Γ $\Gamma := 0$ RICSAC 9 is a 90 degree (θ_2) collision .

Centroidal Yaw Radii of Gyration, k

$m_1 := \frac{W_1}{g}$ $\quad m_2 := \frac{W_2}{g}$ $\quad k_1 := \sqrt{\frac{I_1}{m_1}} = 3.731\,ft$ $\quad k_2 := \sqrt{\frac{I_2}{m_2}} = 5.097\,ft$ $\quad k_1^2 = 13.923\,ft^2$ $\quad k_2^2 = 25.974\,ft^2$

Center of Mass $m_C := \frac{m_1 \cdot m_2}{m_1 + m_2}$ $\quad v_1 := \sqrt{v_{x1}^2 + v_{y1}^2} = 31.09 \cdot \frac{ft}{s}$ $\quad v_2 := \sqrt{v_{x2}^2 + v_{y2}^2} = 31.09 \cdot \frac{ft}{s}$

Define Sin and Cos functions to work with degrees and not radians

Degrees to radians: $Sin(\theta) := \sin(\theta \cdot deg)$ $\quad Cos(\theta) := \cos(\theta \cdot deg)$ "deg" converts radians to degrees

From the above diagram

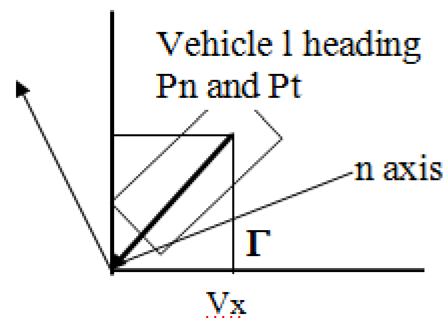
The **common point of collision**, C, is located relative to the C of G by distances d1, d2 and **angles** ϕ_1, ϕ_2. d_a **to** d_d **are** the momentum arms for normal and tangential **Impulse P** (Δp) components.

$d_a := d_2 \cdot Sin(\theta_2 + \phi_2 - \Gamma)$ $\quad d_b := d_2 \cdot Cos(\theta_2 + \phi_2 - \Gamma)$ $\quad d_a = 4.864 \cdot ft$

$d_c := d_1 \cdot Sin(\theta_1 + \phi_1 - \Gamma)$ $\quad d_d := d_1 \cdot Cos(\theta_1 + \phi_1 - \Gamma)$ $\quad d_b = 2.775 \cdot ft$

$d_c = 0.502 \cdot ft$

$d_d = 4.774 \cdot ft$



Vehicle Impulse

Px = Vx - vx, Py = Vy - vy
Pn = Px cos(Γ) + Py sin(Γ))

Given: The Normal and Tangential Velocity Components relative to the Γ normal axis

$v_{n1} := v_{x1} \cdot \cos(\Gamma \cdot deg) - v_{y1} \cdot \sin(\Gamma \cdot deg)$ $\quad v_{n1} = -31.09 \cdot \frac{ft}{s}$

$v_{t1} := v_{x1} \cdot \sin(\Gamma \cdot deg) + v_{y1} \cdot \cos(\Gamma \cdot deg)$ $\quad v_{t1} = 0 \cdot \frac{ft}{s}$

$v_{n2} := v_{x2} \cdot \cos(\Gamma \cdot deg) + v_{y2} \cdot \sin(\Gamma \cdot deg)$ $\quad v_{n2} = 0 \cdot \frac{ft}{s}$

$v_{t2} := v_{x2} \cdot \sin(\Gamma \cdot deg) + v_{y2} \cdot \cos(\Gamma \cdot deg)$ $\quad v_{t2} = 31.09 \cdot \frac{ft}{s}$

Velocity of "C" Common Point Normal Velocity, vcn

Given: The Initial Common/Impact Velocity of Point of Collision point, C.

$v_{cn1} := v_{n1} + d_c \cdot \omega_1 \cdot deg = -31.09 \frac{ft}{s}$ $\quad v_{cn2} := v_{n2} - d_a \cdot \omega_2 \cdot deg = 0 \frac{ft}{s}$

Relative normal velocity vehicle 1 to vehicle 2 Eq. 6.46

$v_{rnx} := (v_{n2} - d_a \cdot \omega_2) - (v_{n1} - d_c \cdot \omega_1) = 31.09 \frac{ft}{s}$ $\quad v_{crn} := v_{n1} + d_c \cdot R \cdot \omega_1 - v_{n2} + d_a \cdot R \cdot \omega_2 = -31.09 \frac{ft}{s}$

$v_{ct1} := v_{t1} - d_d \cdot \omega_1 \cdot deg = 0 \frac{ft}{s}$ $\quad v_{ct2} := v_{t2} + d_b \cdot \omega_2 \cdot deg = 31.09 \frac{ft}{s}$

Given: The Initial Common/Impact Velocity of Point of Collision, C.

$v_{rn} := (v_{n2} - d_a \cdot \omega_2) - (v_{n1} - d_c \cdot \omega_1) = 31.09 \cdot \frac{ft}{s}$

Eq. 6.46

$A := 1 + m_C \cdot \frac{d_c^2}{m_1 \cdot k_1^2} + m_C \cdot \frac{d_a^2}{m_2 \cdot k_2^2} = 1.3$ $\quad B := m_C \cdot \frac{d_c \cdot d_d}{m_1 \cdot k_1^2} + m_C \cdot \frac{d_a \cdot d_b}{m_2 \cdot k_2^2} = 0.282$

Eq. 6.60 to 6.62

$C := m_C \cdot \frac{d_d^2}{m_1 \cdot k_1^2} + m_C \cdot \frac{d_b^2}{m_2 \cdot k_2^2} = 1.214$

Ratio of Initial Tangential to Normal Velocities:

$r = V_{rt}/V_{rn}$

Angle between relative velocity and normal

$r := \frac{v_{ct2} - v_{ct1}}{v_{cn2} - v_{cn1}} = 1$ $\quad \frac{180}{\pi} \operatorname{atan}(r) = 45$

Eq. 6.60 to 6.62

See Pg 6 for Calculation of Optimum ε

Definition: $V_{2n} - V_{1n} = -\varepsilon\,(v_{2n} - v_{1n})$

$\varepsilon := 0.355$ $\beta := 1$

Vcrn = -ε * vcrn (crn=point C, relative, normal)

An important quantity is the **Impulse Ratio, μ,** that gives the **tangential common velocity condition**, that is, where $\mathbf{V_{1Ct} - V_{2Ct} = 0}$. This gives the critical impulse ratio μ_o.

Let β be the fraction of μ_0 that is used for the reconstruction. Then $\mu = \beta * \mu_0$, as shown below.

$\mathbf{P_t = \mu\, P_n}$

$$\mu_{max}(\varepsilon) := \frac{r\cdot A + (1+\varepsilon)\cdot B}{(1+\varepsilon)\cdot(1+C) + r\cdot B}$$ Eq. 6.60

$\mu_{max}(\varepsilon) = 0.512$ $\mu(\varepsilon,\beta) := \beta\cdot\mu_{max}(\varepsilon)$

$\mu(\varepsilon,1) = 0.512$

$d_e(\varepsilon,\beta) := d_c - \mu(\varepsilon,\beta)\cdot d_d$ $d_f(\varepsilon,\beta) := d_a - \mu(\varepsilon,\beta)\cdot d_b$ Eq. 6.59

q is the Brach rotation parameter

$$q(\varepsilon,\beta) := (A - \mu(\varepsilon,\beta)\cdot B)^{-1}$$ Eq 6.55

Brach Planar Impact: Given six initial conditions vx, vy, ω for each vehicle plus ε and μ

Results in planar solution/equations for the 8 unknowns: Final Vx, Vy, Ω, P) for each vehicle

Eq. 6.48 to 6.53

$$V_{n1}(\varepsilon,\beta) := v_{n1} + \frac{m_C\cdot(1+\varepsilon)\cdot v_{rn}\cdot q(\varepsilon,\beta)}{m_1}$$ $$V_{t1}(\varepsilon,\beta) := v_{t1} + \frac{\mu(\varepsilon,\beta)\cdot m_C\cdot(1+\varepsilon)\cdot v_{rn}\cdot q(\varepsilon,\beta)}{m_1}$$

$$V_{n2}(\varepsilon,\beta) := v_{n2} - \frac{m_C\cdot(1+\varepsilon)\cdot v_{rn}\cdot q(\varepsilon,\beta)}{m_2}$$ $$V_{t2}(\varepsilon,\beta) := v_{t2} - \frac{\mu(\varepsilon,\beta)\cdot m_C\cdot(1+\varepsilon)\cdot v_{rn}\cdot q(\varepsilon,\beta)}{m_2}$$

$$V_1(\varepsilon,\beta) := \sqrt{V_{n1}(\varepsilon,\beta)^2 + V_{t1}(\varepsilon,\beta)^2}$$ $$V_2(\varepsilon,\beta) := \sqrt{V_{n2}(\varepsilon,\beta)^2 + V_{t2}(\varepsilon,\beta)^2}$$

$V_1(\varepsilon,1) = 9.669\cdot mph$ $V_2(\varepsilon,1) = 18.885\cdot mph$

Eq. 6.52 and 6.53

$$\Omega_1(\varepsilon,\beta) := \omega_1 + m_C\cdot(1+\varepsilon)v_{rn}\cdot d_e(\varepsilon,\beta)\cdot\frac{q(\varepsilon,\beta)\cdot 180}{\pi\cdot I_1}$$ $\Omega_1(\varepsilon,1) = -199.702\,\frac{1}{s}$

$$\Omega_2(\varepsilon,\beta) := \omega_2 + m_C\cdot(1+\varepsilon)v_{rn}\cdot d_f(\varepsilon,\beta)\cdot\frac{q(\varepsilon,\beta)\cdot 180}{\pi\cdot m_2\cdot k_2^{\,2}}$$ $\Omega_2(\varepsilon,1) = 87.322\,\frac{1}{s}$

Planar Impact (Rigid Body Impact) Mechanics

Impact ΔV and Impulse

$$\Delta V_1(\varepsilon,\beta) := \sqrt{\left(v_{n1} - V_{n1}(\varepsilon,\beta)\right)^2 + \left(v_{t1} - V_{t1}(\varepsilon,\beta)\right)^2}$$ $\text{Impulse}_1(\varepsilon,\beta) := m_1\cdot\Delta V_1(\varepsilon,\beta)$

$$\Delta V_2(\varepsilon,\beta) := \sqrt{\left(v_{n2} - V_{n2}(\varepsilon,\beta)\right)^2 + \left(v_{t2} - V_{t2}(\varepsilon,\beta)\right)^2}$$ $\text{Impulse}_2(\varepsilon,\beta) := m_2\cdot\Delta V_2(\varepsilon,\beta)$

$P_{n1}(\varepsilon,\beta) := m_1\cdot\left(V_{n1}(\varepsilon,\beta) - v_{n1}\right)$ $P_{t1}(\varepsilon,\beta) := m_1\cdot\left(V_{t1}(\varepsilon,\beta) - v_{t1}\right)$

$P_{n2}(\varepsilon,\beta) := m_2\cdot\left(V_{n2}(\varepsilon,\beta) - v_{n2}\right)$ $P_{t2}(\varepsilon,\beta) := m_2\cdot\left(V_{t2}(\varepsilon,\beta) - v_{t2}\right)$

Velocity of Common Point Normal Velocity, Vcn, and Relative Normal, Vcrn

$V_{cn1}(\varepsilon,\beta) := V_{n1}(\varepsilon,\beta) + d_c\cdot\Omega_1(\varepsilon,\beta)\cdot deg$ Eq. 6.45 $V_{ct1}(\varepsilon,\beta) := V_{t1}(\varepsilon,\beta) - d_d\cdot\Omega_1(\varepsilon,\beta)\cdot deg$

$V_{cn2}(\varepsilon,\beta) := V_{n2}(\varepsilon,\beta) - d_a\cdot\Omega_2(\varepsilon,\beta)\cdot deg$ $V_{ct2}(\varepsilon,\beta) := V_{t2}(\varepsilon,\beta) + d_b\cdot\Omega_2(\varepsilon,\beta)\cdot deg$

Consistency check for ε

$V_{crn}(\varepsilon,\beta) := V_{n1}(\varepsilon,\beta) + d_c\cdot deg\cdot\Omega_1(\varepsilon,\beta) - V_{n2}(\varepsilon,\beta) + d_a\cdot deg\cdot\Omega_2(\varepsilon,\beta)$ $-V_{crn}(\varepsilon,\beta)\cdot v_{crn}^{\,-1} = 0.355$

Kinetic Energy Lost by Both and Individual Vehicles

$$KE_{initial} := \frac{1}{2}\cdot\left[m_1\cdot v_1^{\,2} + m_2\cdot v_2^{\,2} + I_1\cdot(\omega_1\cdot deg)^2 + I_2\cdot(\omega_2\cdot deg)^2\right] = 1.074\times 10^5\cdot ft\cdot lb$$

Definition of KE

$$KE_{final}(\varepsilon,\beta) := \frac{1}{2}\cdot\left[m_1\cdot V_1(\varepsilon,\beta)^2 + m_2\cdot V_2(\varepsilon,\beta)^2 + I_1\cdot(\Omega_1(\varepsilon,\beta)\cdot deg)^2 + I_2\cdot(\Omega_2(\varepsilon,\beta)\cdot deg)^2\right]$$

$$\Delta KE_{loss}(\varepsilon,\beta) := KE_{initial} - KE_{final}(\varepsilon,\beta) \qquad Tlo(\varepsilon,\beta) := \frac{1}{2}\cdot m_C\cdot q(\varepsilon,\beta)\cdot v_{rn}^{\,2}\cdot(1+\varepsilon)$$

Total Impact Loss, T_L Eq 6.56

$$T_L(\varepsilon,\beta) := Tlo(\varepsilon,\beta)\cdot\left[2 + 2\cdot\mu(\varepsilon,\beta)\cdot r - (1+\varepsilon)\cdot q(\varepsilon,\beta)\cdot\left[1 + \mu(\varepsilon,\beta)^2 + \left(\frac{m_C\cdot d_e(\varepsilon,\beta)^2}{m_1\cdot k_1^{\,2}} + \frac{m_C\cdot d_f(\varepsilon,\beta)^2}{m_2\cdot k_2^{\,2}}\right)\right]\right]$$

$$\%KELoss(\varepsilon,\beta) := \frac{100(\Delta KE_{loss}(\varepsilon,\beta))}{KE_{initial}} \qquad AngularE_f(\varepsilon,\beta) := \frac{100}{2}\cdot\left[I_1\cdot\left(\Omega_1(\varepsilon,\beta)\cdot\frac{\pi}{180}\right)^2 + I_2\cdot\left(\Omega_2(\varepsilon,\beta)\cdot\frac{\pi}{180}\right)^2\right]$$

$\%KELoss(0,1) = 31.457$ $\qquad \Delta KE_{loss}(0,1) = 3.38\times 10^4\,ft\cdot lb$ $\qquad T_L(0,1) = 3.38\times 10^4\cdot ft\cdot lb$

$$E_{NormalCrush}(\varepsilon,\beta) := P_{n1}(\varepsilon,\beta)\cdot\frac{\left[(V_{cn2}(\varepsilon,\beta) - V_{cn1}(\varepsilon,\beta)) + v_{cn2} - v_{cn1}\right]}{2}$$

$E_{NormalCrush}(0.5,0.5) = 1.412\times 10^4\,ft\cdot lb$

$$E_{TangentialCrush}(\varepsilon,\beta) := P_{t1}(\varepsilon,\beta)\cdot\frac{\left[(V_{ct2}(\varepsilon,\beta) - V_{ct1}(\varepsilon,\beta)) + v_{ct2} - v_{ct1}\right]}{2}$$

$E_{NormalCrush}(0,1) + E_{TangentialCrush}(0,1) = 3.38\times 10^4\,ft\cdot lb$ $\qquad \varepsilon_{PIM}(\varepsilon,\beta) := \sqrt{\dfrac{E_{NormalCrush}(\varepsilon,\beta)}{T_L(0,\beta)}}$

$PCETan(\varepsilon,\beta) := 100\cdot E_{TangentialCrush}(\varepsilon,\beta)\cdot\%KELoss(\varepsilon,\beta)^{-1}$ $\qquad \Gamma_o := \Gamma\cdot\dfrac{\pi}{180}$

Calculate Final X,Y Velocities and Momenta from Γ & Final Normal and Tangential Velocities

$V_{x1}(\varepsilon,\beta) := V_{n1}(\varepsilon,\beta)\cdot\cos(\Gamma_o) - V_{t1}(\varepsilon,\beta)\cdot\sin(\Gamma_o)$ $\qquad V_{y1}(\varepsilon,\beta) := V_{n1}(\varepsilon,\beta)\cdot\sin(\Gamma_o) + V_{t1}(\varepsilon,\beta)\cdot\cos(\Gamma_o)$

$V_{x2}(\varepsilon,\beta) := V_{n2}(\varepsilon,\beta)\cdot\cos(\Gamma_o) - V_{t2}(\varepsilon,\beta)\cdot\sin(\Gamma_o)$ $\qquad V_{y2}(\varepsilon,\beta) := V_{n2}(\varepsilon,\beta)\cdot\sin(\Gamma_o) + V_{t2}(\varepsilon,\beta)\cdot\cos(\Gamma_o)$

$P_{x1}(\varepsilon,\beta) := m_1\cdot(V_{x1}(\varepsilon,\beta) - v_{x1})$ $\qquad P_{y1}(\varepsilon,\beta) := m_1\cdot(V_{y1}(\varepsilon,\beta) - v_{y1})$

$P_{x1}(\varepsilon,\beta) = 1749.993\,s\cdot lb$ $\qquad P_{y1}(\varepsilon,\beta) = 896.544\,s\cdot lb$

$V_{x1}(\varepsilon,\beta) = -6.126\dfrac{ft}{s}$ $\quad V_{y1}(\varepsilon,\beta) = 12.79\dfrac{ft}{s}$ $\quad V_{x2}(\varepsilon,\beta) = -11.498\dfrac{ft}{s}$ $\quad V_{y2}(\varepsilon,\beta) = 25.199\dfrac{ft}{s}$

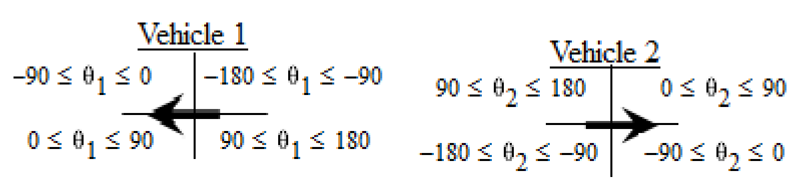
Calculate PDOF RICSAC 9 is a 90 degree collision. Angles are CCW. See drawing on pg. 2.

Sign Conventions: https://www.brachengineering.com/content/vcrware-samples/planar-impact-mechanics.pdf

The Principal Direction of Force, PDOF is the Line of Action of the Total Impulse

$$AT(\varepsilon,\beta) := \frac{180}{\pi}\cdot atan\left(\left|\frac{P_{y1}(\varepsilon,\beta)}{P_{x1}(\varepsilon,\beta)}\right|\right)$$

$AT(\varepsilon,\beta) = 27.127$



$t1 := if(\theta_1 \geq 0, \theta_1, \theta_1 + 360)$ $\qquad t2 := if(\theta_2 \geq 0, \theta_2, \theta_2 + 360)$

$PDOF1(\varepsilon,\beta) := if[(t1 - PF1(\varepsilon,\beta)) > 180, t1 - PF1(\varepsilon,\beta) - 360, IFP1(\varepsilon,\beta)]$

$PDOF2(\varepsilon,\beta) := if(180 + PF2(\varepsilon,\beta) \leq -180, 180 + PF2(\varepsilon,\beta) + 360, IFP2(\varepsilon,\beta))$

$PDOF1(\varepsilon,1) = -27.127$ $\qquad PDOF2(\varepsilon,1) = 62.873$ $\qquad PDOF1(\varepsilon,1) - PDOF2(\varepsilon,1) = -90$

Brach Planar Collision Model Assumptions:

There is a common crush plane at a point, C.
The time duration of an Impact is small, thus changes in mass centers and angles are small.
Collision forces much greater than any of the other forces, e.g. friction, and very short.
Use Impulse and conservation of momentum --> no integration is necessary.
Preimpact dimensions are used.   Changes in angular momentum equal to moments of impulses.
Newton's equations not enough information. Must use normal and tangential impact processes, ε & μ
Find Six unknowns, Given 4 eqs. Two Constraints: Coefficients of Restitution, ε and Impulse Ratio,
Therefore, to get post velocities, we must have knowledge of ε and an estimate of μ

Find Optimal Solution: Least Error for V & Ω

Found by a Least Square Fit between the known Final RICSAC Velocities and Calculated Values

Known Final Velocities for RICSAC 9

$V_{xk1} := -5.1\frac{ft}{s}$ $V_{yk1} := 17.6\frac{ft}{s}$ $\Omega_{k1} := -180\cdot\frac{1}{s}$

$V_{xk2} := -7.3\frac{ft}{s}$ $V_{yk2} := 20.2\frac{ft}{s}$ $\Omega_{k2} := 45\cdot\frac{1}{s}$

Define Least Squares Error Functions, V_{error} (ε) and Ω_{error}(β)

$$V_{error}(\varepsilon x, \beta) := \left[\left(V_{x1}(\varepsilon x, \beta) - V_{xk1}\right)^2 + \left(V_{y1}(\varepsilon x, \beta) - V_{yk1}\right)^2 + \left[\left(V_{x2}(\varepsilon x, \beta) - V_{xk2}\right)^2 + \left(V_{y2}(\varepsilon x, \beta) - V_{yk2}\right)^2\right]\right]$$

Guess $\varepsilon x := 0.1$ $\beta := 1$

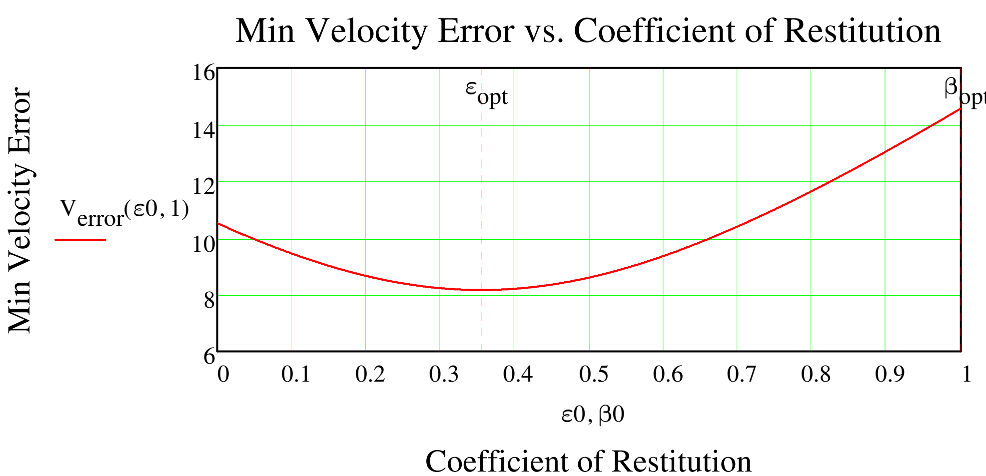
Constraints

Given $\varepsilon x \ge 0$ $\beta \le 1$

Use Mathcad Minimize() Function to Find Values of εopt and βopt for Minimum V & Ω Error

$\varepsilon\beta_{opt2} := \text{Minimize}(V_{error}, \varepsilon x, \beta)$ $\varepsilon\beta_{opt2} = \begin{pmatrix} 0.355 \\ 1 \end{pmatrix}$ $\varepsilon_{opt} := \varepsilon\beta_{opt2_0}$ $\beta_{opt} := \varepsilon\beta_{opt2_1}$

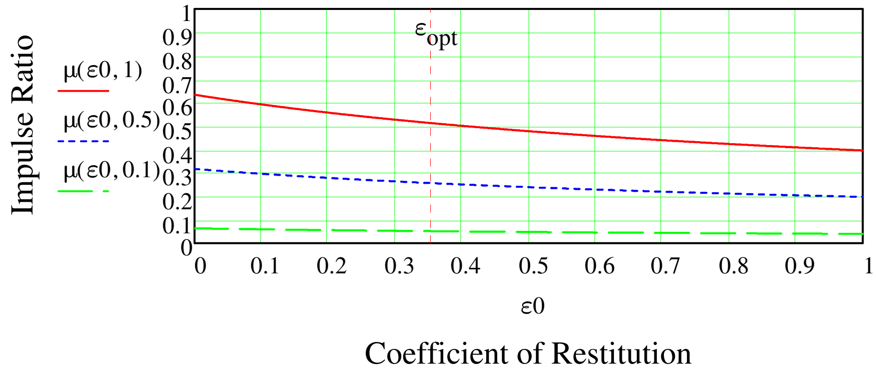
Plot Least Squares Error to Find Optimum ε (εopt) and βopt for Minimum V and Ω Error



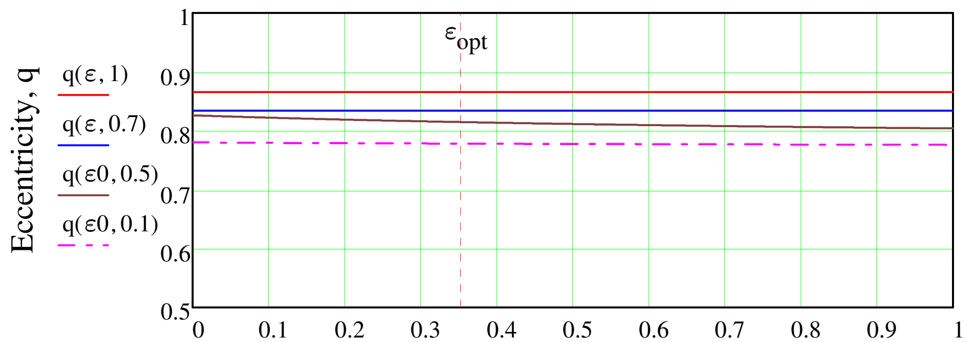
Plot Variation: Impulse Ratio, Eccentricity, Impulse, PDOF, Angular Velocity, & %KE Loss

<u>Versus ε for Various Values of β (1, 0.7, 0.5, 0.1)</u>

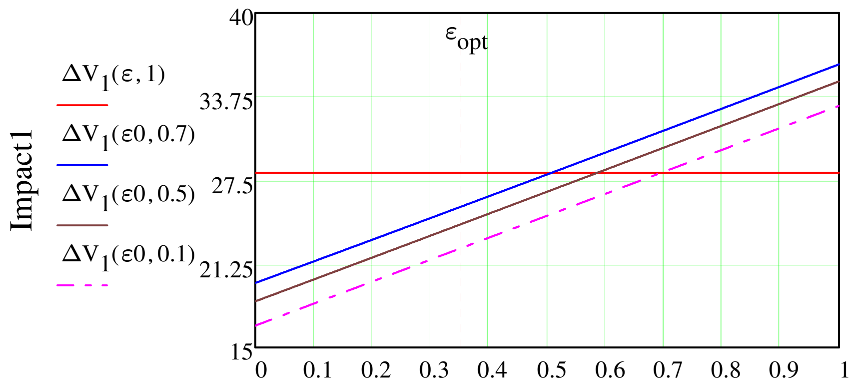
Impulse Ratio vs. Coefficient of Restitution



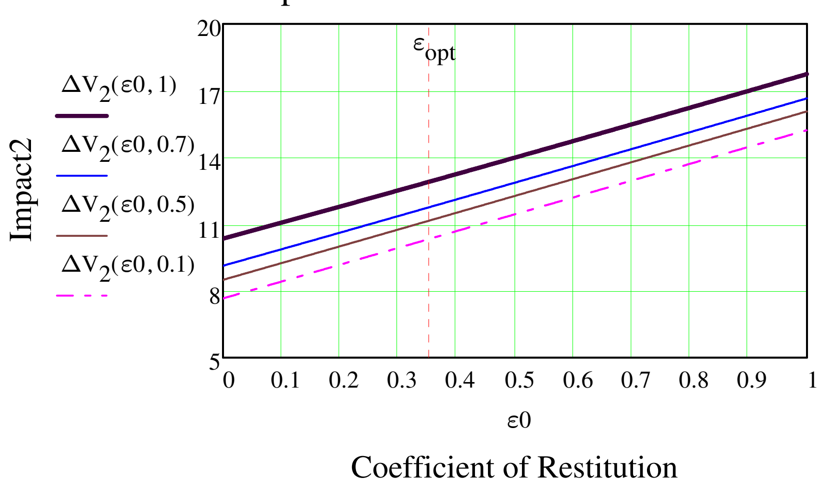
Brach Eccentricity q vs. Coefficient of Restitution

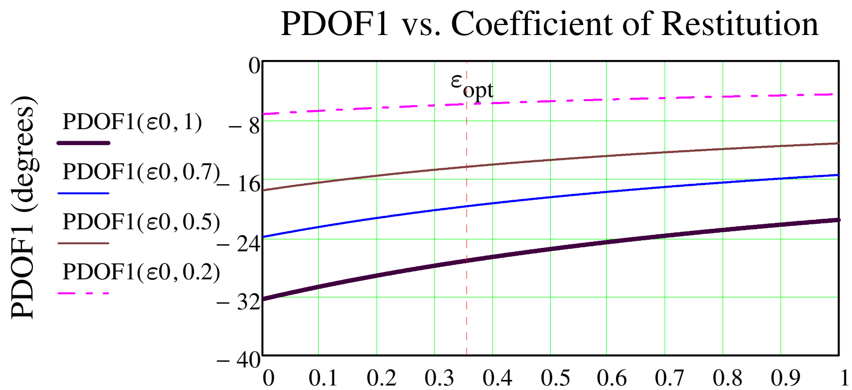


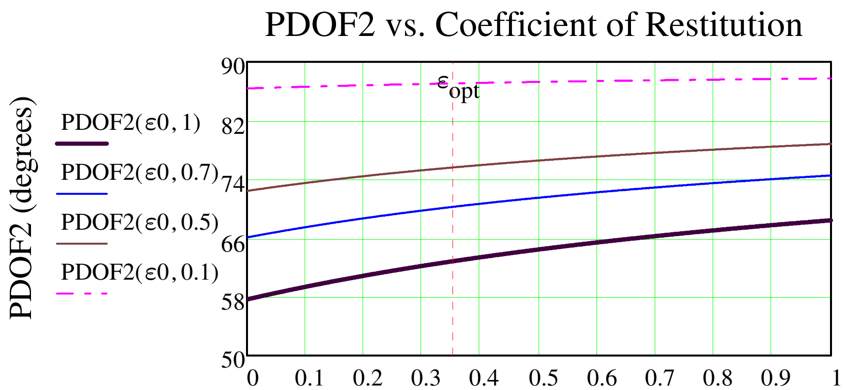
Impact1 vs. Coefficient of Restitution

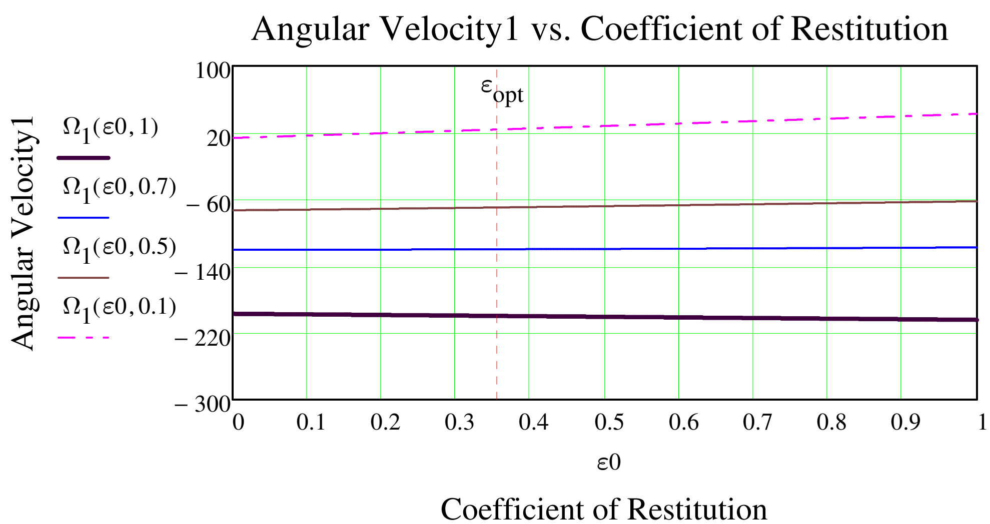


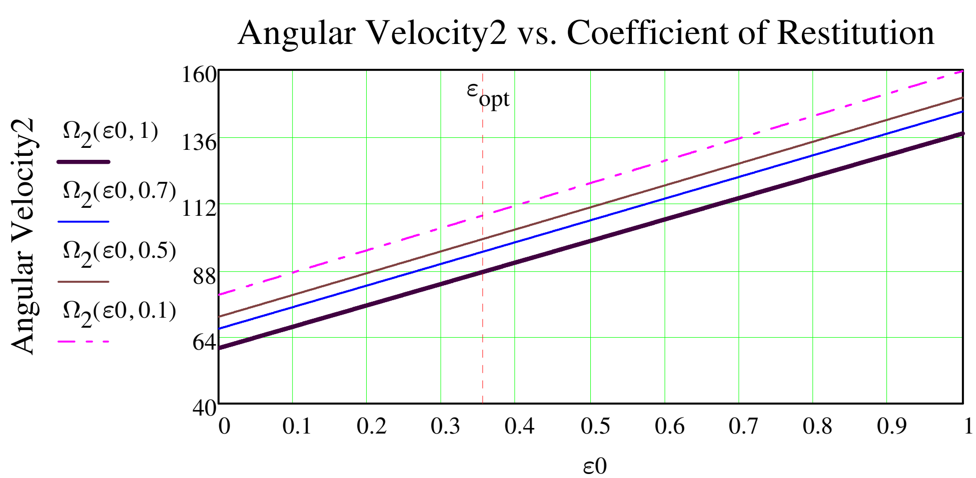
Impact2 vs. Coefficient of Restitution



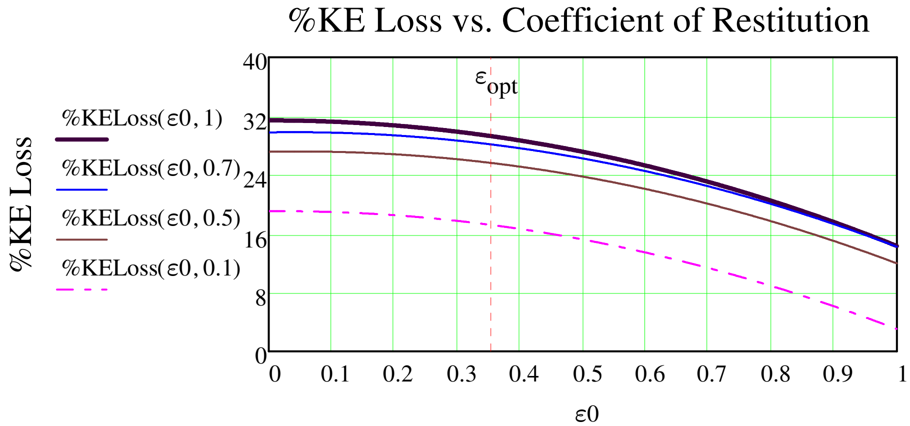

PDOF1 vs. Coefficient of Restitution
PDOF1 (degrees)
PDOF1(ε0, 1)
PDOF1(ε0, 0.7)
PDOF1(ε0, 0.5)
PDOF1(ε0, 0.2)
εopt
0
− 8
− 16
− 24
− 32
− 40
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

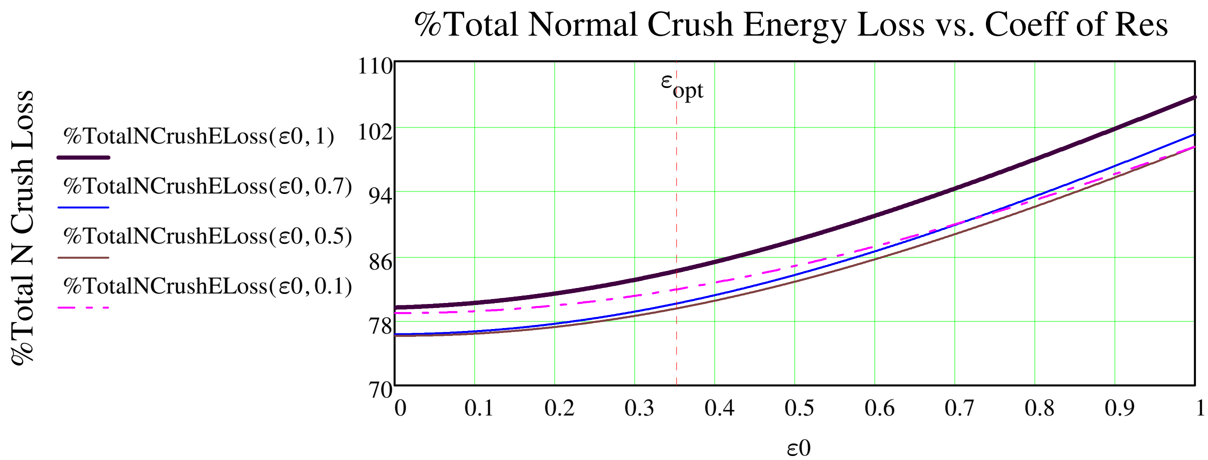

PDOF2 vs. Coefficient of Restitution
PDOF2 (degrees)
PDOF2(ε0, 1)
PDOF2(ε0, 0.7)
PDOF2(ε0, 0.5)
PDOF2(ε0, 0.1)
εopt
90
82
74
66
58
50
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Angular Velocity1 vs. Coefficient of Restitution
Angular Velocity1
Ω1(ε0, 1)
Ω1(ε0, 0.7)
Ω1(ε0, 0.5)
Ω1(ε0, 0.1)
εopt
100
20
− 60
− 140
− 220
− 300
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
ε0
Coefficient of Restitution


Angular Velocity2 vs. Coefficient of Restitution
Angular Velocity2
Ω2(ε0, 1)
Ω2(ε0, 0.7)
Ω2(ε0, 0.5)
Ω2(ε0, 0.1)
εopt
160
136
112
88
64
40
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
ε0

Note: In a Planar Collision a Coefficient of Restitution of 0 can still result in After Collision KE, because of Final Angular Momentum Ω and Tangential Impulse.





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